2.6 Notes and Examples

Name: Block: Seat:

Related Rates

Translation From English to Calculus Notation

1. Units are ______________ important in related rates problems.
2. Here "rate of change" would refer to ______________ rate of change, otherwise known as a

   ______________, unless average rate is mentioned specifically.
3. There will be multiple relationships that can be expressed as an ______________.
4. Many times we use substitution to ______________ a variable in an equation that has more than one variable. We can also use the technique of simultaneous equations where we manipulate and combine a system of equations into a single equation with one variable.
5. We then differentiate with respect to ______________.

Matching

1. $\dfrac{dC}{dt} = 2$ inches per day
2. $\dfrac{dS}{dt} = -2\ \dfrac{\text{yds}^2}{\text{min}}$
3. $\dfrac{dA}{dt} = -2\ \dfrac{\text{ft}^2}{\text{sec}}$
4. $\dfrac{dr}{dt} = 2\ \dfrac{\text{cm}}{\text{min}}$
5. $\dfrac{dV}{dt} = -2\ \dfrac{\text{m}^3}{\text{sec}}$

(A) The radius is increasing at a rate of 2 centimeters per minute

(B) the volume is decreasing at a rate of 2 cubic meters per second.

(C) The surface area is decreasing at a rate of 2 square yards per minute

(D) The area is decreasing at a rate of 2 square feet per second.

(E) The circumference is increasing at a rate of 2 inches per day.

1. The area of a circle is increasing at a rate of six square inches per minute.
2. The volume of a cone is decreasing at a rate of 2 cubic feet per second.
3. The population of Mos Eisley is growing at a rate of three thousand people per year.
4. The height of a tree is increasing at a rate of half a foot per year.
5. The water level in my pool is decreasing at a rate of 2 inches per hour.

6. Differentiate with respect to time:

   (a) $A = \pi r^2$

   (b) $a^2 + b^2 = c^2$

   (c) $x^2 + y^2 = 25$

   (d) $V = \pi r^2 h$

   (e) $V = \dfrac{4}{3}\pi r^3$

   (f) $C = 2\pi r$

   (g) $A = \dfrac{bh}{2}$

   (h) $S = 2\pi rh + 2\pi r^2$
   Hint: Factor out the constants first

5 Steps to Related Rates Problems

1. What are you given? (think of units)
2. What are you asked for? (think of units)
3. What equation(s) connect these?
4. Implicitly Differentiate $\left(\dfrac{d}{dt}\right)$
5. Evaluate @ known value (think of units)

Examples

1. The sides of a square are increasing at a rate of 5 cm/sec. How fast is the area increasing when the sides measure 15 cm in length?

2. A 6 meter ladder is against a wall. The top end of the ladder is sliding down the wall as the bottom is being moved. When the top end is 5 meters from the ground it is being pulled away from the wall at a rate of 1/2 meter per second. How fast is the ladder top sliding when the top of the ladder is 5 meters from the ground?

3. A 10 meter ladder is against a wall. The top end of the ladder is sliding down the wall. When the top end is 6 meters from the ground it is sliding down at 2 meters per second. How fast is the bottom moving away from the wall at this moment?

4. A spherical Balloon is being inflated at a rate of 10 cubic centimeters per second. Find the rate of change of the surface Area of the Balloon at the moment when the surface area is 64π square centimeters.

5. Joe is standing 6 miles due east of Moe. If Joe walks due North at 3 mph while Moe walks due south at 1 mph, at what rate is the distance between them changing after 2 hours?

6. Water is drained out of a full conical tank that measures 10 feet across the top and 12 feet deep. If the water is draining out at the rate of 10 cubic feet per minute, what is the rate of change of the depth of water when the depth is 8 feet?

7. A 6 feet tall man walks away from a 16 foot street light at a rate of 2 feet per second. As he walks, the shadow lengthens. When he is 8 feet from the streetlight, what is the rate at which the shadow is lengthening?

Answers

(1) The area is increasing at rate of 150 square centimeters per second when the side reaches 15 cm in length
(2) The ladder is sliding down the wall at a rate of about 0.332 meters per second.
(3) The bottom of the ladder is moving away from the wall at a rate of 1.5 meters per second when the height of the ladder is 6 meters from the ground
(4) The surface area of the balloon is increasing at the rate of 5 square centimeters per second when the surface area of the balloon is 64π square centimeters
(5) After 2 hours, the distance between Joe and Moe is increasing at a rate of 3.2 mph
(6) The water is draining at a rate of $9/10\pi$ (about 0.286) feet per minute (3.438 inches per minute) when the water depth is at 8 feet.
(7) The length of the shadow is growing at a rate of 1.2 feet per second (or 14.4 inches per second).